A collaboratively-derived research agenda for e-assessment in university mathematics

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ABSTRACT

This paper describes the collaborative development of an agenda for research on e-assessment in university mathematics. We adapted an established approach to develop the agenda from the contributions of 22 mathematics education researchers, university teachers and learning technologists interested in this topic. The resulting set of 55 questions are grouped into 5 broad themes: errors and feedback, students, design and implementation, affordances, and mathematical skills. This agenda gives the framework for a programme of research aligned with practical concerns, that will contribute to both theoretical and practical development.

1. Introduction

Over the past two decades, e-assessment has become widespread in education, with a range of tools available (e.g., Deeva et al., 2021). Specialised e-assessment tools have been developed to meet the particular needs of assessing mathematics (Sangwin, 2013), and the increasing technical sophistication of these tools has underpinned the increasing use of e-assessment in university mathematics (Iannone and Simpson, 2021). This widespread use of e-assessment presents a need for practitioners to be informed by existing research, and conversely for researchers to direct attention at emerging practical concerns. Indeed, Bakker, Cai and Zenger (2021) identified “assessing online” as one of eight challenges for mathematics education research in the coming decade, based on an international survey of mathematics education researchers.

What is needed is a programme of “use-inspired basic research” (Lester, 2005, p. 465), that contributes to both improved theoretical understanding and improved practice. To support such a programme of research, a research

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agenda can set out the important questions that need to be addressed (e.g., [English, 2008](#)). Well-formed research questions thus play a crucial role (e.g., [Cai et al., 2019](#); [Cai and Mamlok-Naaman, 2020](#)).

In this paper, we report on a collaborative project that has established an agenda for such a programme of research. Our approach is inspired by a recent project to establish a research agenda in numerical cognition ([Alcock et al., 2016](#)), that was itself modelled on similar exercises in other fields ([Sutherland et al., 2011](#)). In the next section we describe the process we used to achieve a similar outcome for e-assessment of university mathematics. In the remainder of the paper we present the resulting set of 55 questions for future research in this area.

2. Method

The project was initiated and led by Kinnear. A core team planned the project ([Kinnear, Jones and Sangwin, 2020b](#)), arranged and chaired meetings, and led on the writing of this manuscript. We used a five-stage Delphi method ([Hsu and Sandford, 2007](#)) in order to develop consensus towards a research agenda. Our plans were based on previous exercises in numerical cognition ([Alcock et al., 2016](#)) and conservation policy ([Sutherland et al., 2011](#)). In our case we were aiming to develop a research agenda within a small community that comprises both researchers and e-assessment system developers, rather than only educational researchers.

We identified and contacted participants from our existing networks, and through conferences attended by researchers and developers of e-assessment for university mathematics. We generated an initial list of 22 contacts to invite beyond the core team and, following suggestions from these contacts, we sent invitations (see [Appendix A](#)) to a further six people.

We recruited 22 participants as shown in [Table 1](#). Participants were predominantly in Europe and particularly the UK, reflecting the fact that the core research team is based in the UK.

Stage 1: Gathering questions. We used an online survey to gather an initial set of research questions. Participants were advised that questions should focus on formative or summative assessment of university mathematics relying on technology in a fundamental way. We also asked participants to provide a short motivation for each question, and to describe what an answer might look like. This enabled participants to put the question in context, since “the significance of a research question cannot be determined just by reading it. Rather, its significance stands in relation to the knowledge of the field” ([Cai et al., 2019](#), p. 118).

Participants proposed a total of 61 questions, with between 1 and 9 questions per participant and a mode of 2.

Stage 2: Prioritising. Following [Alcock et al. \(2016\)](#) we planned to identify the questions deemed most important by participants using an online rating system. However, as we had a relatively small ratio of questions (61) to participants (22) we decided this stage was not necessary, and retained all questions.

Stage 3: Refining questions. We held a working group meeting, originally planned to be in-person but forced online by the Covid-19 pandemic, in which participants discussed the questions in order to suggest refinements and to identify connections between them. Prior to the meeting, the core research team divided participants and their proposed research questions into three groups. The groups were constituted so that each was chaired by one member of the core research team, and each group covered a range of countries, affiliations and interests. Participants were then asked to prepare for the meeting by reading all the questions from their assigned group in detail, and also browsing the questions from other groups to note any connections (see [Appendix B](#) for details).

The online meeting lasted two hours and was attended by 21 of the participants listed in [Table 1](#). During the meeting the participants worked within their groups, with each group facilitated by a member of the core project team, and were asked to do the following:

- Clarify the questions by adjusting wording, or removing/combining/splitting questions as needed.
- Identify connections aiming to identify possible themes or groupings.

Some participants also submitted written comments on the questions before or after the meeting.

Following the meeting, the core team met to consolidate the suggested refinements. Some of the original questions were deleted, while others were split into two or even three distinct questions. This resulted in 52 questions. So that these could be presented in a structured way to a wider audience in the following stage, the core team arranged the

Participant	Country	Affiliation	Questions submitted (Stage 1)	Questions edited (Stage 5)
André Heck	The Netherlands	University of Amsterdam	3	3
Ann O'Shea	Ireland	Maynooth University	3	3
Ben Davies	UK	UCL	2	4
Chris Sangwin	UK	University of Edinburgh	1	3
Colin Foster	UK	Loughborough University	3	5
George Kinnear	UK	University of Edinburgh	4	16
Ian Jones	UK	Loughborough University	5	10
Igor' Kontorovich	New Zealand	The University of Auckland	3	5
Indunil Sikurajapathi	UK	University of the West of England	0	4
John Meyer	UK	University of Birmingham	0	2
Karen Henderson	UK	University of the West of England	5	4
Maryam Alarfaj	UK	University of Edinburgh	2	1
Morten Brekke	Norway	University of Agder	1	0
Niclas Larson	Norway	University of Agder	4	13
Paola Iannone	UK	Loughborough University	2	2
Pat Barmby	UK	No More Marking Ltd.	2	0
Peter Rowlett	UK	Sheffield Hallam University	9	26
Rhys Gwynllyw	UK	University of the West of England	4	0
Sam Fearn	UK	Durham University	0	1
Thomas Wong	UK	Heriot-Watt University	2	2
Tim Hunt	UK	The Open University	1	2
Tim Lowe	UK	The Open University	5	13

Table 1

Participant locations, affiliations and number of questions submitted at Stage 1. Note that some participants submitted no questions at Stage 1 since they joined later in the process.

questions into five sets of approximately equal size, using the connections identified by participants to try to keep related questions in the same set. These sets formed a basis for – but were ultimately different from – the themes presented later in this manuscript.

Stage 4: Gathering feedback. The activities described for Stage 3 were repeated, minus the preparation work, at three online conferences (E-Assessment in Mathematical Sciences, June 2020; Mathematics Education in the Digital Age, September 2020; 4th Northeastern RUME Conference, October 2020). At each conference, the core team presented an overview of the project activities, before sharing the sets of questions and inviting participants to leave feedback using an online survey. This enabled input from a broader range of participants (including internationally) in order to increase the validity and buy-in from the wider community. At this stage, we focused on refining the existing questions rather than adding more questions, although we did invite participants to suggest where we might have gaps. We received seven proposals for new questions; upon review by the core team, four were deemed to duplicate existing questions and the remaining three were added for the following stage.

Stage 5: Finalising the agenda. We planned to use conference participants' feedback from Stage 4 to further focus the agenda. However, few participants completed the survey asking for their priorities; the responses we did receive were widely distributed across the questions. As a result, we decided not to attempt to prioritise the list of questions. Instead, we retained the full set of 55 questions, and invited the working group to populate a website (<https://maths.github.io/e-assessment-research-agenda/>) that would host details about each question. Most participants in Table 1 attended a 2-hour online meeting in January 2021 to begin the process of populating the website, following the guidance shown in Appendix C. During this meeting, small discussion groups formed organically to discuss questions related to particular topics, and to start making plans for future collaboration on those questions. Editing of the website continued asynchronously over the following months alongside preparation of this manuscript. Members of the working group added their name against those questions that they contributed to fleshing out on the website, with the number of questions for each member shown in Table 1.

3. Research agenda for e-assessment in university mathematics

Before presenting the research questions, we note some decisions on consistent use of terminology throughout the manuscript. We use:

- *e-assessment* to mean the use of computer technology to deliver assessment. This encompasses uses that may be referred to elsewhere as *Computer-Aided Assessment* (CAA) or *Computer-Aided Instruction* (CAI).
- *teacher* or *lecturer* to refer to the person making decisions about the use of e-assessment as part of teaching.
- *task designer* to refer to the person creating e-assessment tasks, which can be a distinct role that does not involve any in-person teaching. The task designer could be the lecturer or may be a learning technologist or even a student intern.
- *student* to refer to the learners who may be using e-assessment.
- *university mathematics* as the assumed context for teaching, learning and assessment throughout.

The 55 questions in the research agenda are arranged here in clusters of up to five tightly related questions, and these clusters are grouped into five broad themes. The themes and clusters were initially formed by Kinnear following a thematic analysis approach (Braun and Clarke, 2006), with minor adjustments made through discussion among the core team during the process of finalising this manuscript. These themes are for coherence of presentation and do not reflect a hierarchy or intended prioritisation of research questions.

3.1. Errors and feedback

One of the strongest themes is around detecting and responding to student errors. Feedback has long been identified as an educationally important intervention (Kluger and DeNisi, 1996), and there is advice in the general education literature on how to maximise its effectiveness (Shute, 2008). Many e-assessment systems can provide automated feedback, which responds to students' actual answers, in a way which is intended to improve their performance on the particular task (Sangwin, 2013, p.33–35). There are a wide range of features that can be varied, such as the timing and content of feedback. Scholars have been investigating these variations (e.g., Attali and van der Kleij, 2017) but many questions remain unresolved.

3.1.1. Student errors

1. What common errors do students make when answering online assessment questions?
2. Do the errors students make in e-assessments differ from those they make in paper-based assessments?
3. What are the approaches to detecting and feeding back on students' errors?

Mathematics education has a well-established research tradition of analysing students' work for common errors (e.g., Hart, 1987). The findings are then used to develop our understanding of mathematical learning, and to improve pedagogy and resource design. Since e-assessment systems typically store all student responses, there is a large set of historic and continuously generated data to be mined worldwide. Researchers are therefore currently well-positioned to contribute to the literature on student errors (e.g., Sikurajapathi, Henderson and Gwynllwy, 2020).

The validity and scope of such research would require addressing Question 2 to identify errors that arise specifically in e-assessments as opposed to paper-based assessments. In the former, there is often no requirement for students to show their solution method, and student errors can arise due to technology (clicking the wrong button or slips in syntax) that we would not expect in paper-based assessments. We might hope students routinely have a calculator, pencil and paper to hand, but this needs to be investigated (see also Question 13).

Question 3 relates to and precedes the research questions in section 3.1.2. For example, upon detecting an error we might need to discern whether it is a common student error, or due to a misreading of the question or a miscalculation. This could be a challenging task across different randomised variants of the same question.

3.1.2. Feedback design

4. *How can content-specific features of provided feedback, for instance explanations with examples versus generic explanations, support students' learning?*
5. *What are the linguistic features of feedback that help students engage with and use feedback in an online mathematical task at hand and in future mathematical activities?*
6. *What difficulties appear when designing e-assessment tasks that give constructive feedback to students?*

The design of feedback is a longstanding concern, both practically and for researchers. This group of questions addresses design issues highlighted by reviews of existing research on formative feedback (e.g., [Shute, 2008](#); [Palm et al., 2017](#)).

Two of the questions have a particular focus on the design of elaborated feedback. First, Question 4 addresses a specific concern about the content of elaborated feedback: whether it should be based on generic explanations or should highlight particular examples. For instance, in a question about adding fractions, the feedback could describe in general terms the process (“find a common denominator”, etc.) or could present a particular worked example; the two approaches may differ in the way they support students' learning. Second, Question 5 addresses the linguistic features of elaborated feedback, such as the appropriate level of detail and technical language to use. Investigation of this question may involve experimental comparisons of different approaches, but could also build on existing qualitative work that explored students' interpretations of written feedback on proofs ([Byrne et al., 2018](#)).

More generally, Question 6 is concerned with exploring features that make feedback more constructive. The focus is on the difficulties encountered by task designers in implementing such feedback (this is related to questions about task design in [section 3.3](#)). One feature that might be considered here is the distinction between explicit and implicit feedback ([Devlin, 2011](#)). For example a question asking for a quadratic with given roots could produce an explicit message saying whether the answer is right or wrong, or a graph of the function supplied by the student that implicitly conveys whether the answer is correct.

3.1.3. Emulating teacher feedback

7. *How can feedback that is dynamically tailored to the student's level of mathematical expertise help a student use feedback on mathematical tasks effectively?*
8. *How useful for students' long-term learning is feedback that gives a series of follow-up questions, from a decision tree, versus a single terminal piece of feedback?*
9. *What are the relative benefits of e-assessment giving feedback on a student's set of responses (e.g. “two of these answers are wrong – find which ones and correct them”), rather than individual responses separately?*

This group of questions are all motivated by a desire to use e-assessment to emulate features of teachers' approaches. Despite the fact that previous researchers and developers working on intelligent tutoring systems “totally abandoned our original conception of [automatic] tutoring as human emulation” ([Anderson et al., 1995](#), p. 202), it is likely that there are some aspects of the human teacher's approach to feedback that could be fruitful in e-assessment.

Prior knowledge about students can influence the approach to giving feedback, and Question 7 asks how e-assessment might tailor feedback to best effect. An underlying motivation for this question is the expertise reversal effect ([Kalyuga et al., 2003](#)), which suggests that the relative effectiveness of different types of feedback may depend on a student's level of task-specific expertise.

Question 8 asks whether a more dialogic approach to feedback could be effective in e-assessment. The suggestion is that, at each stage, the feedback given to the student should be the minimum that is necessary to direct their attention (i.e. a scaffolding and fading approach; see [Mason, 2000](#); [Foster, 2014](#)). Similarly, Question 9 considers the possibility that making feedback deliberately less specific may be advantageous, as it may encourage students “to look at the feedback more closely and to think about their original work more analytically” ([William, 2016](#)).

3.1.4. Optimising feedback efforts

10. *Under what circumstances is diagnosing errors worth the extra effort, as compared with generally addressing errors known to be typical?*

11. *What are the relative merits of addressing student errors up-front in the teaching compared with using e-assessment to detect and give feedback on errors after they are made?*
12. *In what circumstances is instant feedback from automated marking preferable to marking by hand?*

A particular concern here is the effort required to detect and give appropriate feedback on specific errors, since this may be nontrivial (see [section 3.1.1](#)). Question 10 asks whether it may be more efficient to give only generic feedback, such as a model solution. There are conflicting findings on this point in the literature (e.g., [Rønning, 2017](#); [Attali and van der Kleij, 2017](#)), so the focus of this question is on identifying circumstances where the diagnostic effort is worthwhile.

In a similar vein, Question 11 asks whether the effort used for diagnosis and feedback could be better spent on developing up-front teaching that addresses common student errors. [Foster et al. \(2021, p. 12\)](#) characterise this distinction as “diagnose and treat” versus “treat all”, and favour the “treat all” approach in their work on curriculum design. Finally, Question 12 addresses a tension between immediacy and quality of feedback. Instant feedback is often argued to be beneficial because it corrects any errors while the student is still thinking about the question, however it has also been noted that automated feedback may not be sufficiently responsive to students’ learning needs ([Broughton, Hernandez-Martinez and Robinson, 2017](#); [Rønning, 2017](#)).

3.2. Students

Many of the research questions that arose from this project can be related to students’ interactions with e-assessment. The questions in this group have an explicit concern with how students perceive and respond to e-assessments. We know from the general education literature that there is a link between students’ perceptions of the demands of learning and their engagement with learning ([Marton and Säljö, 1997](#)). Accordingly, the questions straddle aspects of sociocultural, cognitive and design issues.

3.2.1. Student behaviour

13. *How do students interact with an e-assessment system?*
14. *To what extent does repeated practice on randomized e-assessment tasks encourage mathematics students to discover deep links between ideas?*
15. *How do students engage with automated feedback? What differences (if any) can be identified with how they would respond to feedback from a teacher?*
16. *What should students be encouraged to do following success in e-assessment?*

The focus on students’ interactions and learning is explicit in these four questions. Question 13 is concerned with how students behave, questions 14 and 16 relate to designing for optimal learning, and Question 15 considers both behaviour and design.

Question 13 is expressed in general terms and some relevant studies have already been carried out. [Dorko \(2020\)](#) used video recordings of students undertaking online homework along with follow-up interviews and reported that feedback and multiple tries offered by the online system contributed to students working iteratively when solving problems. Further research should investigate the robustness of [Dorko’s](#) findings across different technologies, cohorts and contexts. Other aspects of student behaviour, such as social interactions when using e-assessment systems, should also be considered (see [section 3.2.3](#)).

The last three questions might be effectively addressed through intervention studies, possibly incorporating experimental designs involving control groups. Question 14 can be contrasted with current work in applying cognitive science to mathematics practice, such as interleaving ([Rohrer, Dedrick and Stershic, 2015](#)) and spacing ([Lyle et al., 2020](#)). The questions in this group also have connections with the questions in [section 3.1](#), but offer a different perspective. For instance, Question 15 contrasts with the assumptions of the questions in [section 3.1](#) which are concerned with the minutiae of design. Instead, Question 15 focuses on the potential of a computer to replace a teacher; a timely issue given increasing cohort sizes and remote learning.

3.2.2. *Student views and outcomes*

17. *What are students' views on e-assessment, and what are their expectations from automated feedback?*
18. *How might dyslexic, dyscalculic and other groups of students be disadvantaged by online assessments rather than paper-based assessments?*

These questions take a broader view of students' interaction with e-assessment than those in the previous section.

Question 17 asks about students' views on e-assessment. The way that e-assessment is used in a course will clearly be an important factor influencing students' views, so answers to this question should pay careful attention to the context. For instance, [Rønning \(2017\)](#) found that where e-assessments were compulsory but unlimited attempts were allowed, students viewed the e-assessment as a process of "hunting for the answer". It would also be worthwhile to understand students' views in relation to other forms of assessment – particularly given that mathematics undergraduates tend to express a preference for more traditional forms of assessment ([Iannone and Simpson, 2015](#)).

Question 18 is concerned with the potential for differential outcomes for various groups of students based on their use of e-assessment. For instance, it could be the case that dyslexic students face additional barriers when using e-assessment, and therefore perform less well than might be expected. There is a notable lack of research on this topic, given the increasingly widespread use of e-assessment, and it has been noted elsewhere as an "important question for the field" ([Cai et al., 2020](#), p. 525).

3.2.3. *Student interactions*

19. *How can peer assessment be used as part of e-assessment?*
20. *How can e-assessment be used in group work, and what effect does the group element have on individuals' learning?*

Peer assessment is trumpeted for potential benefits such as generating more student feedback than can be achieved by a lecturer ([Topping, 2009](#)), and promoting learning through students viewing one another's work ([Jones and Alcock, 2014](#)). Research on peer assessment in higher education increasingly involves technology (e.g., [Ashenafi, 2017](#)), although mathematics-specific e-assessment technologies tend not to overtly support peer learning and assessment activities. Studies addressing Question 19 are likely to focus on generic peer assessment technologies that can be used for mathematics, see for example [section 3.4.3](#).

Nevertheless, students sometimes spontaneously form support groups when taking online tests ([Alcock et al., 2020](#)). Lecturers may wish to encourage students to work in groups to harness the benefits of peer learning. Question 20 relates to how understanding the way students collaborate when working on e-assessments can help us better understand how to promote collaborations that are productive for learning.

3.3. *Design and implementation*

The questions in this group are primarily concerned with the choices that must be made by the lecturer or task designer, whether at the level of designing an individual e-assessment question, or at the level of integrating e-assessment into a coherent course design.

3.3.1. *Task design principles*

21. *What design methodologies and principles are used by e-assessment task designers?*
22. *What principles should inform the design of e-assessment tasks?*
23. *E-assessment task designers often convert questions that could be asked on a traditional pen and paper exam: what are the implications, technicalities, affordances and drawbacks of this approach?*

Design principles are often implicit in task designers' practice, and Question 21 seeks to make them explicit. Examples of design principles being made explicit in previous work include Sangwin's (2013, Chapter 3) description of general principles of assessment design, and Kinnear et al.'s (2021) account of the specific principles that informed the design of an online course built around e-assessment. An answer to Question 21 could be based on a systematic review of such literature. It may also be informed by work on Question 23, which highlights the common practice of converting or "translating" existing paper-based tasks to e-assessment. This approach may have implications for the range of tasks

that are set using e-assessment—some existing paper-based tasks may be “untranslatable”, while other tasks that would be suited to e-assessment may not be considered.

Question 22 goes beyond asking what principles are currently in use, and seeks to identify the principles that *should* be used. This could be informed by work on Question 21 (and others, e.g. from [section 3.1](#)) to identify possibilities, as well as drawing on the expertise of e-assessment task designers.

3.3.2. Randomisation

24. *To what extent does the randomisation of question parameters, which makes sharing answers between students difficult, adequately address plagiarism?*
25. *What effect does the use of random versions of a question (e.g. using parameterised values) have on the outcomes of e-assessment?*
26. *When writing multiple choice questions, is student learning better enhanced using distractors based on common errors, or randomly-generated distractors?*

The ability to randomly generate versions of questions is a core feature of many mathematics e-assessment systems and “creates opportunities not present with a fixed paper worksheet” ([Sangwin, 2003](#), p. 38). One of the justifications given for the importance of generating different versions of a question for each student is that it mitigates against plagiarism. Question 24 asks whether this is really the case, and suggests replicating and extending quantitative work that seeks to detect the possible extent of plagiarism, with a particular focus on whether the use of randomisation is seen to have an effect.

Question 25 asks about the effect of randomisation on outcomes more generally. A particular concern here is the effort required to devise and test randomised questions, and whether this is justified by the suggested benefits (such as giving students the opportunity to repeatedly practise key skills). The issue of authoring effort is also relevant in Question 26, which focuses on multiple choice questions. A standard recommendation to authors of multiple choice questions is to base distractors on common student errors (e.g., [Gierl et al., 2017](#)). Given the difficulty that task designers face in anticipating such student errors (see [section 3.1.1](#)), it could be more efficient to use randomly-generated distractors for questions with numerical answers; there is also the possibility that students who make an error could benefit from what is effectively immediate feedback when they do not see their answer listed as an option.

3.3.3. Role of e-assessment in course design

27. *How can formative e-assessments improve students’ performance in later assessments?*
28. *How can regular summative e-assessments support learning?*
29. *What are suitable roles for e-assessment in formative and summative assessment?*
30. *To what extent does the timing and frequency of e-assessments during a course affect student learning?*
31. *What are the relations between the mode of course instruction and students’ performance and activity in e-assessment?*

These questions are all concerned with decisions about how e-assessment can be used within a course, such as the timing and incentives associated with completing the assessments.

One important course design decision is whether e-assessments are used formatively or summatively. Formative e-assessments are commonly used to provide students with opportunities to practise skills, consistent with the view that “mathematics needs to be *done* to be *learned*” ([Greenhow, 2015](#), p. 120). However, the extent to which students engage with these formative assessments is variable and, in some cases, time spent by students on e-assessment can be detrimental to their overall performance ([Hannah, James and Williams, 2014](#)). Thus, Question 27 is concerned with identifying ways that e-assessment can be employed formatively to best effect. Summative e-assessments can range from a small portion of the course grade, through to forming the basis of the entire course (e.g., [Sangwin and Kinnear, 2021](#)). Question 28 asks how such uses can support learning, and the suggested outcome is a collection of case studies of different models, together with some evaluation of their effectiveness.

Question 29 ties together the questions 27 and 28, and asks how both formative and summative assessments can be used as part of course design. Each approach may be suitable for achieving different aims, and this is closely related

to the capabilities of the e-assessment tool (see also [section 3.4](#)). For instance, current tools are perhaps best-suited to assessment of procedural skills, and this informs the ways that e-assessment can be used as part of course design.

Another course design decision is about the frequency and timing of e-assessment. For instance, some courses make extensive use of e-assessment with weekly (or even more frequent) tasks for students to complete (e.g., [Kinnear et al., 2021](#); [Heck et al., 2022](#)). Question 30 invites a comparison of different approaches, such as between the use of shorter more frequent e-assessments throughout a course, and fewer e-assessments (e.g. at the end of substantial topics).

Finally, Question 31 seeks to understand how different approaches to course design influence the way that students interact with, and perform in, e-assessment as part of the course. For instance, there is now a large body of research showing that active learning approaches are more effective than traditional lecturing ([Freeman et al., 2014](#)), including studies of mathematics teaching (e.g., [Maciejewski, 2015](#)), but e-assessment has not been prominent in this research so far.

3.3.4. Lecturer guidance

32. *What advice and guidance (both practical and pedagogical) is available to lecturers about using e-assessment in their courses, and to what extent do they engage with it?*
33. *What might a “hierarchy of needs” look like for lecturers who are transitioning to increased use of e-assessments?*
34. *How can lecturers be informed about how students interact with e-assessment tasks, and so help lecturers act upon these findings in an effective way?*

Moving from analysis of student interactions (see [section 3.2.1](#)) to implications for practitioners is not straightforward ([Alcock et al., 2020](#)), and addressing these research questions might require a programme of research projects.

Question 32 arose from a context in which lecturers had to create materials from scratch and come up with questions of their own. Analysis of such materials along with methods such as interviewing authors in such cases would be informative. Question 33 might be answered using the same methods and case studies, with analysis focussing on identifying the steps, skills, equipment, expertise and so on required to construct good e-assessments.

Question 34 focuses not on *a priori* design principles but monitoring and responding to students’ interactions. Some systems offer data on student interactions, for example systems such as STACK or SOWISO provide data on student responses and results ([Sangwin, 2013](#); [Rienties et al., 2019](#)), but this can be overwhelming so it is not always clear how lecturers might act in response.

3.4. Affordances

These questions are about the features of existing e-assessment tools and the relationships of these tools to assessment objectives. The research questions consider what is possible, the extent to which this fulfils various purposes, and possible ways the features could be extended. There is a subtle interplay between the assessment format, questions we ask students and the roles we discussed in [Section 3.3.3](#). This has always been the case and is not only an issue for e-assessment, for instance, paper-based multiple choice questions require careful design ([Gierl et al., 2017](#)) and using them to assess extended forms of reasoning is difficult.

3.4.1. Capabilities of e-assessment

35. *What types of reasoning are required to complete current e-assessments?*
36. *To what extent do existing e-assessments provide reliable measures of mathematical understanding, as might otherwise be measured by traditional exams?*
37. *How can e-assessment support take-home open-book examinations?*
38. *What developments at the forefront of e-assessment (such as artificial intelligence) can we apply to university mathematics?*

These questions, as a group, form a natural sequence, starting with question design. [Lithner \(2008\)](#) developed a framework for classifying the types of reasoning required by mathematical tasks. Other similar task classification schemes were proposed by [Smith et al. \(1996\)](#) and [Pointon and Sangwin \(2003\)](#). Such classification schemes have been used for research (e.g., [Darlington, 2014](#)) and more informally as aids to the design of formative assessments.

Such frameworks enable the study of the types of tasks assigned in university level courses (Kinneer et al., 2020a; Mac an Bhaird et al., 2017), but it seems that very little work has been done in the area of e-assessment.

Questions 36 and 37 relate to examinations. Traditional examinations have a number of advantages. Examinations place a highly constrained time limit on the assessment (in a way more open project work does not), and we can be reasonably confident that impersonation of the candidate does not take place. The controlled conditions within a traditional examination allow explicit choices to be enforced, such as the availability of books and other reference materials, and the use of technology such as calculators. There is also the social experience of the “event” when turning up at a traditional examination venue en-mass with peers, indeed perhaps even a right of passage. There are efforts to replicate traditional exams online (e.g., Sangwin and Köcher, 2016), and it is important to understand how these relate to traditional exams (notwithstanding criticism of traditional examinations). Some of the issues associated with examinations have nothing to do with mathematics as a particular subject, but these questions focus on the issues that are mathematics-specific. Further questions related to moving from traditional approaches to e-assessment appear in section 3.4.2.

Finally, Question 38 looks to the future, and how new developments in e-assessment technology can be applied in university mathematics. These developments include advances in interpreting free-form input (taken up further in section 3.4.2), and the use of artificial intelligence to augment or replace human judgement in assessments.

3.4.2. Free-form student input

39. *What methods are available for student input of mathematics?*
40. *How can the suitability of e-assessment tools for summative assessment be improved by combining computer-marking and pen-marking?*
41. *Are there differences in performance on mathematics problems presented on paper versus as e-assessments?*
42. *How can we automate the assessment of work traditionally done using paper and pen?*
43. *How can we emulate human marking of students' working such as follow-on marking and partially correct marking?*

These questions all relate to automatic assessment of complete mathematical arguments. Assessment of students' free-form input probably remains one of the most significant challenges in e-assessment, both from the technical perspective of software design, and the complexity of the traditional assessment process.

A central problem is how students might input their argument into a computer (Question 39). Clearly one option is to photograph handwriting and upload it, perhaps in addition to keying in a final answer; this sort of hybrid approach is considered in Question 40. However, most of the questions tacitly assume moving beyond merely uploading.

One of the significant barriers faced by mathematics students interacting with e-assessment is the heavy use of special symbolism. This motivates Question 41, since the use of special syntax is one factor that may lead to a difference in performance in e-assessment compared with paper. For example, Sangwin and Ramsden (2007) found that a substantial source of student errors in one e-assessment system were due to difficulties in using the particular “linear syntax” for entering expressions. Notational ambiguity occurs between mathematical sub-disciplines, unsurprisingly leaving potential for serious confusion about the interpretation and meaning of students' work (see for example Kontorovich and Zazkis (2017)). All this said, e-assessment provides an opportunity to explore and research students' intended meaning precisely because the syntax is often rather strict.

Question 42 seeks to understand the extent to which we automate the assessment of mathematical work traditionally done using paper and pen. Most professional mathematicians typeset their work with systems like LaTeX. Unfortunately LaTeX is rather difficult to learn and the typesetting process can interrupt the train of thought. Students are likely to confound authoring with typesetting when using a computer to type their work. While e-assessment is likely to establish acceptance from the mathematics community by replicating traditional pen and paper assessment, there is nothing to suppose that replication is desirable or effective. Beyond issues of notation, there may be richer and more open-ended assessment tasks which might benefit from automated question-setting (say, for randomisation) but which generate unstructured student work that is difficult to mark automatically. A system where the questions are generated by an e-assessment system, but the resultant submissions are marked by hand, is explored by Rowlett (2020). A hybrid system, where part of the submission is marked automatically and passed to an intelligent human marker, may also be possible.

Question 43 considers the marking of students' line-by-line working, and there have been many attempts to provide students with constrained interfaces to help students work line by line. There are many examples from MathXpert (Beeson, 1998) through to SOWISO (Heck, 2017). At this stage, engineering of software is far ahead of serious evaluation of the individual decisions made by the software designers, or the effectiveness of such software. Understanding how to automatically assess free-form student input, or mix automated assessment with human assessment underpin all the questions in this section.

3.4.3. Comparative judgement

44. *How can comparative judgement be used for e-assessment?*

45. *How can e-assessment using comparative judgment support learning?*

Comparative judgement offers a method of grading students' work that involves making holistic judgements about the relative quality of students' work without reference to a rubric (Pollitt, 2012). These holistic pairwise decisions are then used to derive a score for each piece of students' work, and these scores can then be used for ranking or grading purposes as required. Comparative judgement has received attention in mathematics education over the past decade due to its promise as a reliable and valid method for assessing important but nebulous learning outcomes such as conceptual understanding (e.g., Jones and Alcock, 2014), proof comprehension (e.g., Davies, Alcock and Jones, 2020) and problem solving (e.g., Jones and Inglis, 2015). Higher education e-assessment tends to focus on mathematical accuracy and consequently comparative judgement, with its focus on genuinely open-ended test questions, can help broaden students' assessment diet. Question 44 considers issues of using comparative judgement for assessment, whereas Question 45 considers student learning, although in practice both questions are likely to be intertwined and addressable through iterative design research methods (McKenney and Reeves, 2018).

The potential barriers to lecturers using comparative judgement for summative assessment include comparative judgement's lack of rubrics, meaning students might not know what they are aiming for, and its lack of traditional 'red ink' feedback; for some higher education institutions these aspects may be unacceptable. Nevertheless comparative judgement can and has been implemented in higher education institutions (e.g., Jones and Sirl, 2017), and the potential barriers for summative assessment can be strengths for student learning. For example, through structured peer assessment activities (see also Question 19 in section 3.3) students, rather than lecturers, can judge one another's responses to open-ended test questions to identify what constitutes a high-quality response in the absence of rubrics. Moreover, feedback can be reconceptualised in terms of students comparing one another's responses with reference to their own, rather than in terms of 'red ink' received from the teacher or e-assessment system.

3.5. Mathematical skills

While the whole research agenda is specific to university mathematics, the questions in this theme are particularly directed at how e-assessment can be used to assess and develop mathematical skills: problem solving, proving and generating examples. Numerous works have discussed the nature of the subject and the skills mathematics education might seek to develop (e.g., Polya, 1954; Freudenthal, 1973; Mason, Burton and Stacey, 2010). E-assessment has, mostly, been used to provide formative feedback for skill-based practice, often in calculus and algebra. Existing e-assessment tools, and contemporary know-how in using them, start to become less useful for the more challenging questions raised in this section. However, our experience suggests that e-assessment can be applied in some areas to good effect, and further work may identify opportunities and where the boundaries of effective e-assessment really lie.

3.5.1. Problem solving

46. *How can we assess problem solving using e-assessment?*

47. *How can we assess open-ended tasks using e-assessment?*

48. *How can e-assessments provide scaffolding (cues, hints) during and after problem-solving tasks?*

These three questions are closely related. Answers to these questions have been among the goals of the earliest pioneers of e-assessment, who worked on what was often titled *intelligent tutoring systems* or *computer-aided instruction*. By the 1980s these rather ambitious goals were being reassessed (see Sleeman and Brown, 1982, Preface). One common approach has been to break up larger problem solving tasks into smaller individual questions to which

e-assessment can then be applied. The e-assessment system then builds up a model of the state of the students' knowledge (e.g. Appleby, Samuels and Jones, 1997). A common conclusion is that a significant investment of time and experience is needed for a good network of questions and related skills (Anderson et al., 1995). There have been many attempts at providing hints, clues and feedback. Some of these attempts are based largely on mathematical concerns, other attempts on models of how students learn. Feedback to students varies between specific feedback designed to help students with a particular task through to more general feedback indicating to an individual student which skills they have mastered.

3.5.2. *Assessment of proof*

49. *How can the assessment of proof be automated?*
50. *What can automated theorem provers (e.g. LEAN) offer to the e-assessment of proof comprehension?*
51. *What types/forms of proof-comprehension-related questions can be meaningfully assessed using currently available e-assessment platforms?*
52. *How can students effectively type free-form proof for human marking online?*

Proof is the hallmark that distinguishes mathematics from other subjects (Hanna, 1983). However, the meaning of “assessment of proof” is open to interpretation and has a number of subtly interrelated aspects. Deciding when a proof is correct is far from easy, since teachers have differing expectations on the level of detail required. What constitutes an acceptable size of gap in the reasoning for the particular class differs significantly between subject areas; for instance, logic and foundation courses might require that students make much more specific reference to axioms and particular deduction rules than a traditional calculus methods course. Both contain proofs, but they look very different. Such variety is a significant challenge in general, although it is certainly possible to assess proof in particular constrained areas of mathematics using specially-designed tools.

That said, mathematical proof may (or may not) be a more constrained and structured language than other subject areas. This is perhaps where the use of automated theorem provers have a role. By their very nature automatic theorem proving software constrains users to the syntax and norms of the prover. This makes the proof checking possible, but also creates a significant barrier to use (Avigad, 2019). Learning to use the theorem prover becomes as much a part of the process as learning the mathematics we are trying to prove (e.g., Thoma and Iannone, 2021). The use of theorem provers has not been widely accepted within many sub-disciplines of the mathematics community, and asking students to learn to use a theorem prover may not be an acceptable path for many colleagues. However automated theorem provers are attracting some interest in teaching first year students as a way to develop programming skills.

One promising area in the short term is not assessment of free-form proof, but rather assessment of aspects related to proof. A number of aspects were suggested, but not evaluated, by Sangwin and Bickerton (2021), including proof comprehension which was discussed in detail by Mejia-Ramos et al. (2017). Davies et al. (2020) considered “proof summaries”, a promising idea, and used comparative judgement (see section 3.4.3) for e-assessment.

3.5.3. *Example generation*

53. *How can e-assessments be designed to expand and enrich students' example spaces?*
54. *To what extent can e-assessments meaningfully judge student responses to example generation tasks?*
55. *How does the use of e-assessment impact students' example generation strategies and success, relative to the same tasks on paper or orally?*

These questions allude to research suggesting that example generation tasks can encourage students to engage with new concepts, and to expand their awareness of representations and instances of those concepts (Watson and Mason, 2006). However, feeding back on student-generated examples can be a difficult task for teachers, and is impractical with large cohorts. As such, e-assessment could play a powerful role in providing students with formative feedback on their examples (Sangwin, 2003). Questions 53 and 54 might involve investigating how existing, paper-based tasks can be implemented in e-assessment systems and, importantly, how we can reliably check the properties of students' inputted examples. For example, if asking for functions which tend to 0 at infinity, the computer algebra system would need to have the ability to evaluate limits of arbitrary functions.

Question 55 is concerned with evaluating the educational benefits of such approaches, using paper-based example generation tasks as a control. Limitations to automating the feedback of example generation tasks are likely to be encountered; for example, students may immediately be able to sketch an example with the required properties, or orally describe the relevant features, but it is not obvious how such responses can be inputted in order to provide automated feedback. (This relates to the theme of ‘free form student input’ in e-assessment; see [section 3.4.2](#).)

4. Discussion

Through a collaborative process, we have developed a shared agenda for research on e-assessment of university mathematics. The agenda consists of 55 research questions that we have grouped into 5 broad themes: errors and feedback, students, design and implementation, affordances, and mathematical skills. The wide range of questions underscores the complexity of research on learning, with topics ranging from issues of design and implementation, though to cognitive, social, and sociocultural issues. Research to advance this agenda will therefore benefit from drawing on a range of theoretical perspectives ([Lester, 2005](#)).

The agenda’s broad range of questions invite a range of research approaches. Some of the questions could be addressed by literature reviews or analysis of existing data. For instance, methodologies and principles used by e-assessment task designers (Question 21) could be identified by reviewing literature (e.g., [Greenhow, 2015](#); [Sangwin, 2013](#); [Sangwin and Bickerton, 2021](#); [Heck et al., 2022](#)). Other questions would benefit from qualitative approaches, such as interviewing students to better understand their views on e-assessment (Question 17), or observing students as they work on tasks to gain insight into their processes (e.g. example generation strategies in Question 55). Many of the questions would be best addressed using experimental methods, which have been relatively under-used in the field of mathematics education in recent years ([Alcock, Gilmore and Inglis, 2013](#); [Inglis and Foster, 2018](#)). For instance, questions about novel approaches to feedback ([section 3.1.3](#)) could naturally be addressed with randomised controlled trials comparing student outcomes under different feedback conditions. Moreover, some questions could be suitable for a multi-site approach, similar to the recent *ManyClasses* study on the efficacy of delayed feedback ([Fyfe et al., 2021](#)). For instance, comparing different approaches to timing of assessments (Question 30) across many contexts would enable a better understanding of possible moderators of their effectiveness.

We have not made any attempt to prioritise questions in the agenda. Previous exercises have included a round of prioritising, to reduce the original list of questions submitted by participants to those deemed most important (e.g., [Alcock et al., 2016](#); [Sutherland et al., 2011](#)). While we had originally planned to do this at Stage 2, we decided that this was not necessary since the set of submitted questions was of a manageable size. Instead, we simply consolidated the few submissions that overlapped. We believe this is a pragmatic approach, that has enabled the agenda to address a diverse range of concerns.

The research agenda reflects the interests and concerns of our working group. This could mean that the agenda is not representative of the wider field; for instance, our group is predominantly based in the UK, and many of us share a background in working with a particular set of e-assessment tools. However, our approach of presenting the agenda at various international meetings provided us with feedback on the emerging agenda from different perspectives, and the questions are not specific to any particular platform.

While we do not claim that our agenda is universal, it is clear that many of our concerns are shared by other researchers (e.g., [Bakker et al., 2021](#); [Cai et al., 2020](#)). One notable omission that became apparent near the end of our process is a lack of questions related to diversity, equity and inclusion. We do have one question in this vein (Question 18), but there are clearly other important questions that could have been added. For instance, one of the participants in the survey by [Bakker et al. \(2021\)](#) asked: “What roles could digital technology play, and in what forms, in restoring justice and celebrating diversity?”

One of our aims in carrying out this exercise was to stimulate new research collaborations to address these questions. This has already had some success, with collaborations underway to address various questions from the agenda. Members of the group would welcome further collaboration with the wider community. To facilitate this, the project website (<https://maths.github.io/e-assessment-research-agenda/>) shows which group members have particular interest in each of the questions. This site will be updated as progress is made on addressing the questions.

We also aimed to offer a method to the computers and education community, in which researchers and developers often overlap and work together, for setting research agendas. The focus here has been specific – e-assessment in university mathematics – but the method could readily be applied to other areas of e-learning and indeed to education research and development more widely.

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A. Invitation email

I am writing to invite you to collaborate on a new working group that aims to develop a shared research agenda for computer-aided assessment of university mathematics. Such an agenda will help to establish a programme of research aligned with practical concerns, which would contribute to both theoretical and practical development. The working group is being run by George Kinnear, Chris Sangwin and Ian Jones.

We will follow methods used to develop a research agenda in numerical cognition, see [Alcock et al. \(2016\)](#).

We will begin with an online phase to gather and prioritise an initial set of questions. This will be followed by a series of in-person meetings to discuss and further prioritise the questions. Throughout the process input from the broader research community all also be sought. All collaborators will be invited to contribute to a scientific paper to publish the agenda in a peer reviewed outlet. For reference, I've attached a short paper that we've written explaining the project in more detail. [*the attachment was a copy of Kinnear et al. (2020b)*]

At this stage we are seeking three things from you. First, a reply to this email to let us know if you are interested or not. Second, suggestions for colleagues we could also contact. Third, completion of an online survey to gather the initial set of questions as described above. The survey can be found at <https://edinburgh.onlinesurveys.ac.uk/questions-in-online-assessment>

We hope you are able to give serious consideration to this request. We'd love to have you on board!

B. Instructions ahead of Stage 3 meetings

- “Read all the questions from your group. Note any queries or comments you can bring to the discussion for instance:
 - are there statements that could be clarified?

- do you know of any existing related research?
- do you have thoughts on the possible approaches to answering the question?
- Look at questions from the other groups and note any connections with the questions that you proposed. This will speed up the process of us identifying overlapping concerns, while distributing the effort.”

C. Instructions on editing questions

This repository gives us a way to collaborate on writing about the set of research questions.
As a contributor, you should:

1. make sure you are happy with your profile,
2. add yourself as a contributor on any questions you are interested in,
3. edit the pages of those questions, to add/amend details.

Most questions only have the text of the question so far. Please help to flesh out the details!
This can be informed by:

- the original submission – see the document from our first meeting which has the original proposer’s full narrative,
- comments from the survey/conferences – you can see all of these in the files in Google Drive,
- your own further thoughts/reading.

Adding related questions

Many of the questions are connected, and we’d like to highlight this. There is a section on each question page for "Related questions" which should end up with a list of points about how the question connects with others.